Dynamical gain saturation in Kerr-lens mode-locked CW solid-state lasers

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It is shown that dynamical gain saturation, usually neglected for solid-state lasers, can contribute significantly to the pulse-shaping mechanism in Kerr-lens mode-locked lasers. In particular, in combination with self-phase modulation it can cause red shifts and blue shifts of the laser spectrum. A control of the reciprocal gain and loss saturation rates using these shifts leads to improved mode-locking stability.

Gain saturation (GS) strongly affects the mode-locking dynamics because it contributes to the competition between high- and low-intensity field spikes. In the presence of loss saturation, this leads to the formation of a solitary pulse, which saturates the gain and suppresses the laser noise [1]. Moreover, as will be demonstrated here, dynamical gain saturation is also an important pulse-shaping factor. It can contribute to the mode-locking dynamics in the sub-100 fs region [2] and would be even more important for picosecond lasers. In combination with self-phase modulation (SPM), GS can cause a shift of the pulse carrier frequency from the center of the gain band. Since modern mode-locking techniques, e.g. Kerr-lens mode-locking [3], make use of a nonlinear phase response, the influence of GS in the presence of SPM should be taken into account. Furthermore, as was shown in [4], the combined action of SPM and GS can form a soliton-like pulse, which can be stabilized by some additional mode-locking mechanism.

Here we present a detailed investigation of the influence of GS on the characteristics of both sub-100 fs and ps pulses in Kerr-lens mode-locked lasers. We show that, unlike in the case of slow loss saturation [4], the frequency shift of the pulse caused by SPM can be used to stabilize the laser output and decreases the sensitivity of the pulse duration to group-velocity dispersion (GVD).

It is known that is solid-state lasers dynamical gain saturation, i.e. the gain saturation by the laser pulses during one round-trip, is small. Typically, the pulse energy is three orders of magnitude lower than the corresponding saturation energy. For example, for the Nd : YAG the saturation fluence is \( \approx 0.4 \text{ J cm}^{-2} \), while energy fluence of the pulse is \( \approx 2 \text{ mJ cm}^{-2} \) [5]. However, an important laser parameter that significantly contributes to pulse shaping is \( \tau = \sigma_{22} \tau_0 / \varepsilon \beta \), where \( \sigma_{22} \) is the gain cross-section, \( \varepsilon \) the photon energy, and \( \tau_0 \) the pulse duration.
the inverse spectral filter bandwidth, and $\beta$ the SPM coefficient. The factor $\tau$ is of order of one and thus should be taken into account.

Within a self-consistent approach, the evolution of the field propagating through the system can be described as follows \[4, 6, 7\]:

$$\frac{\partial a(k + \delta, t)}{\partial k} = [\alpha - \gamma + i\phi - \pi \int_{k}^{t} |a|^2 \, dt' + (1 + id) \frac{\partial^2}{\partial t^2} + (1 - i)|a|^2]a(k, t)$$  \hspace{1cm} (1)

where $k$ is the round-trip number, $t$ the local time, $\alpha$ the gain, and $\gamma$ the loss. The time and the phase delay of one cavity round-trip are denoted by $\delta$ and $\phi$, respectively. The terms $\pi \int_{k}^{t} |a|^2 \, dt'$ and $|a|^2$ describe GS and the absorber-like action due to Kerr-self-focusing, respectively. The term $-i|a|^2$ takes SPM into account. Spectral filtering and GVD are accounted for by the term $(1 + id)\partial^2/\partial t^2$, where $d$ is the GVD coefficient. We normalized the intensity on the saturation intensity of the effective Kerr-lens saturable absorber $\sigma = \beta$. All times in Equation 1 are normalized to $t_0$, and the GVD coefficient $d$ to $t_0^{-2}$.

Equation 1 has a soliton-like solution in the following form \[6, 8\]:

$$a(t) = a_0 \text{sech}^{1+i\phi}[(t - k\delta)/t_p]\exp[i\omega(t - k\delta)]$$  \hspace{1cm} (2)

where $a_0$ is the pulse amplitude, $t_p$ the duration normalized on $t_0$, $\psi$ the chirp, $\omega$ the shift of the pulse carrier frequency with respect to the spectral filter band normalized on $t_0^{-1}$.

After substitution of Equation 2 into Equation 1 we obtain a system of the six nonlinear algebraic equations. Their solution is:

$$\omega = \left[2\alpha(1 + d^2)(1 + d)(3d - 3 - \sqrt{17d^2 - 2d + 17}\tau)\right]^{1/2} \left[9 + 7d^2 - 17d - 19d^3 + 10 + \sqrt{17d^2 - 2d + 17}(5 + 5d^2 - 2d)\right]^{1/2}$$ \hspace{1cm} (3a)

$$\delta = -\frac{\omega}{2(d + 1)} [3 + 4d^2 + d \pm \sqrt{17d^2 - 2d + 17}]$$ \hspace{1cm} (3b)

$$t_p^2 = \frac{1 - \frac{4\omega}{\delta + 2d\omega}}{\omega^2 + \gamma - \alpha}$$ \hspace{1cm} (3c)

$$\psi = \frac{2\omega}{\delta + 2d\omega}$$ \hspace{1cm} (3d)

$$\alpha^2 = (\alpha - \gamma - \omega^2)\left(\delta^3 + 6\delta^2d\omega + 4\delta d^2\omega^2 + 12\delta^2d^2\omega^2 + 8d^3\omega^3 \right.$$ \hspace{1cm} (3e)

$$\left. + 8d^3\omega^3\right)/[\pi\omega(4\omega^2 + 4d^2\omega^2 - \delta^2)]$$

$$\phi = (\alpha\delta^2d - \delta^2d\gamma + 4\alpha\delta d\omega + \delta^2\omega + 4\delta d^2\omega - 4\delta d\gamma \omega - 4\delta^2\gamma \omega$$ \hspace{1cm} (3f)

$$+ 4\delta d\omega^2 + 4\delta d^2\omega^2 - 4\delta d\gamma \omega^2 - 4d^3\gamma \omega^2 - 8\delta d\omega^3 - 8\delta d^2\omega^3 - 8d^3\omega^3 - 8d^2\omega^3)$$

Figure 1 shows the region where solutions of Equation 1 in form (2) exist. Here, $g = \alpha - \gamma$ is the net gain at the pulse maximum. The region is divided into three subregions: The first has an arbitrary GVD and negative net gain(subregion 1, Fig.1). In the second, both GVD and $\tau$ are different from zero, and the net gain is positive (subregion 2 of Fig. 1). In the third, the pulse exists inside a narrow domain of $g, d$ and $\tau$ (subregion 3 in Fig. 1). The last type of solution is unstable, and we shall not consider it further.
The dependence of the frequency shift $\omega$ and the pulse duration $t_p$ on GVD $d$ for the first type of solution is shown in Fig. 2a, b. The same dependence for the second type of solution is presented in Fig. 2c, d. The chirp, which is nearly the same for both solutions, is shown in Fig. 2e.

It is seen that the dependence of the pulse parameters on $d$ is different for the first and the second type of solution. In the first case (curve 1 corresponds to efficient Kerr-lens mode-locking, when the Ti : Sapphire laser operates close to the boundary of the resonator stability region), the behavior of the pulse duration is similar to the Schrödinger soliton. There is a sharp minimum of $t_p$ for some negative GVD; the pulse duration is longer for positive than for negative GVD. The frequency shift is small for this case and vanishes at the point where the chirp is compensated due to the balance of GVD and SPM. However, this regime differs from the ordinary regime of the Schrödinger soliton formation due to the presence of gain, loss and Kerr-lensing (see [4, 9]).

The transition from the edge of the resonator stability region decreases the Kerr-lens mode-locking efficiency. In our notation, this is equivalent to a decrease of $\sigma$ [10]. This factor and the growth of $t_p$ up to 0.1–1 ps or the use of media with a relatively large gain cross-section cause an increase of $\tau$, i.e. the relative rate of GS. For example, $\tau = 3$ for Nd : YAG laser in the case of low efficient Kerr-lensing, when nonlinear loss coefficient is $10^{-7}$ W$^{-1}$ [10] and $\sigma = 10^{12}$ cm$^2$ W$^{-1}$ for 1000 $\mu$m$^2$ mode cross-section. The growth of $\tau$ causes an increase of the frequency shift (curve 2 in Fig. 2a) and changes the dependence of pulse width on $d$ (curve 2 in Fig. 2b). It is interesting that the growth of $t_p$ due to the rise of $|d|$ becomes slower in comparison with the case of small $\tau$. Here, the pulse durations for positive GVD are close to those produced by negative $d$.

We attribute this behavior of the pulse duration to the shift of the carrier frequency produced by SPM and GS. This shift is caused by predominant amplification of the pulse front, where the lower (higher) frequency components are located for the case of positive(negative) chirp [3]. The influence of this additional pulse-shaping factor on the second

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**Figure 1** Regions of the existence of quasi-solitons: normalized dispersion parameter $d$ versus net gain at the pulse maximum $g$. Region 1 and 3: $\tau = 0.1$. Region 2: $\tau = 1$. 

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type solution is demonstrated by Fig. 2c, d. It is seen that the chirp growth due to rise of $|d|$ (Fig. 2e) decreases the pulse duration and increases the frequency shift for nonzero $\tau$ and positive net gain. The existence of these solutions is due to the balance of two factors: GS and the fact that spectral components located on the pulse front are pushed out of the gain band (see [3, 4]). Such a “nonlinear spectral mode” changes the behavior of the pulse parameters (compare Fig. 2a, b, and Fig. 2c, d).
To investigate the stability of the obtained soliton-like solutions, we used the criterion of pulse stability against small pump energy perturbations $\partial E/\partial P > 0$, where $E$ is the pulse energy and $P$ is the pump energy [11]. The physical explanation for this criterion is as follows: if a small rise of the pump energy increases the pulse energy, it means that the excess energy is concentrated in the pulse. Otherwise, the excess energy will be fed into the noise, tending to decrease the pulse energy and suppress the pulse.

In our case

$$\alpha = \alpha_m \frac{1 - \exp(-P)}{1 - \exp(-P - \tau E)} \exp(-\frac{1}{2} \tau E),$$

where $\alpha_m$ is the maximal gain.

The result of the stability analysis for the first type of solution is presented in Fig. 3. Here, the stability region is plotted in the coordinates pump energy versus net gain. The arrow indicates the direction of a growth of the pulse energy. For relatively small energies, the pulse does not saturate the gain, so the noise does not decay and, as a result, the pulse becomes unstable. The higher pump energy stabilizes the pulse, because it leads to higher pulse energies and gain saturation. However, still further growth of the energy destabilizes the pulse and gives rise to satellites. As one can see from Fig. 3, a large negative GVD or moderate positive GVD raises the pulse stability threshold (the transition from curve 2 to curves 1, 3 or from curve 5 to curves 4, 6). The same effect is produced by an increase of $\tau$ (curves 4–6) due to the gain decrease, which is accompanied by the growth of the gain saturation.

These results were confirmed by numerical simulations, which were based on the fluctuation model [12]. We considered a four-mirror Kerr-lens mode-locked laser with the following parameters: length of the output arm: 114 cm; length of arm with high-reflective mirror: 60 cm; length of the active element (Ti: Sapphire): 0.75 cm. The length of the folding section formed by two spherical mirrors with the radii of curvature of 10 cm was 10.99 cm, and the distance between the active element and spherical mirror was 5.54 cm. A
circular aperture with a diameter of 1 mm was placed at a distance of 1 cm from the output mirror. We assumed that the laser contains an intracavity bandwidth-limiting element with inverse bandwidth \( t_f = 500 \) fs.

For the relatively small pump energy (Fig. 4, curve 1), the pulse duration is minimal for small negative GVD and increases rapidly with \(|d|\). This increase results in a loss of stability due to spreading of the pulse (compare with the increase of the stability threshold in Fig. 3, curves 1 and 3). Higher pump energy leads to a frequency-shifted pulse, because of the increased chirp and gain saturation. In this case, the growth of the pulse duration associated with the increase of \(|d|\) is much weaker, and short pulses exist even for positive GVD (Fig. 4, curve 2). The sign of the frequency shift agrees with analytically predicted sign (anti-Stokes for great negative GVD, and Strokes for positive and small negative GVD).

It was also shown by numerical calculations that the growth of the spectral bandwidth corresponding to the decrease of the GS contribution (greater \( t_f \) and, hence, greater \( \tau \)) prevents generation of short pulses for positive GVD.

The nonlinear frequency shift due to SPM and GS can stabilize the pulse because of the additional nonlinear loss produced by pushing the pulse out of the gain band. The latter prevents the increase of the pulse and the destabilizing influence of SPM.

Figure 5 shows the dependence of the frequency-offset from the center \( \omega \) plotted as a function of \( d \) and pulse intensity \( I = d_0^2 \). It can be seen that for the different operation conditions there are both negative and positive signs of \( \omega \). However, in all cases the frequency shift increases with higher pulse intensity. This produces an additional loss for the high-intensity pulses, an effect that is similar to absorber darkening caused by two-photon absorption. The pulse oscillation decays rapidly in this case, and the system reaches the stability region for finite, physically reasonable energy.

In conclusion, we have shown that the interplay between gain and loss saturation in the presence of SPM and GVD leads to significant changes of the temporal and spectral characteristics of the ultrashort pulses in the picosecond and sub-100 fs time regime. The shift of the pulse carrier frequency from the center of the gain band resulting from the above factors slows down the broadening of the pulses for GVD different from the optimum value. The additional spectral loss due to the nonlinear frequency shift stabilizes ultra-short pulse generation.

![Figure 4](image-url) Pulse duration \( t_p \) versus normalized dispersion \( d \) calculated from a numerical model for normalized pump intensities 0.001 (1) and 0.01 (2). Pump intensity is normalized to \( \varepsilon/T_{cav}r_{14} \), where \( T_{cav} \) is the cavity period, \( r_{14} \) is the active medium absorption cross-section, \( \varepsilon \) is the pump photon energy, \( t_f = 500 \) fs.
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Figure 5 Quasi-soliton frequency shift $\omega$ versus normalized dispersion $d$ and quasi-
soliton peak intensity $I$; $\epsilon = 1$, $\gamma = 0.01$. 